Exam II MTH 213, Spring 2019

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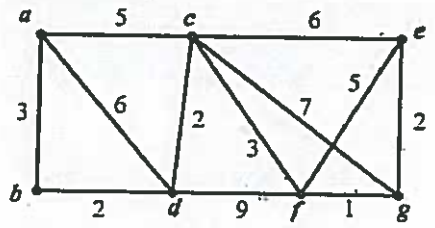
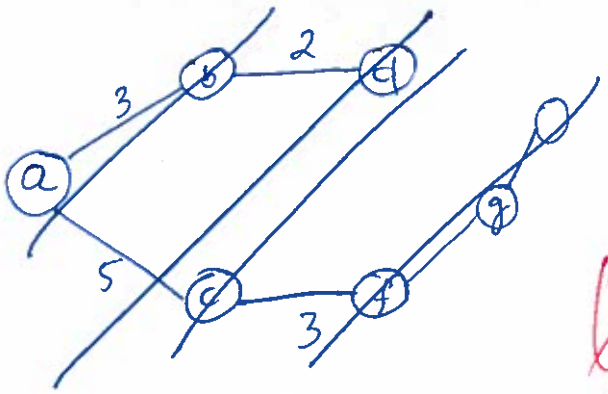
QUESTION 1. Consider the following Graph G. Use Dijkstra's Algorithm and find the minimum spanning tree. Start from the vertex f.

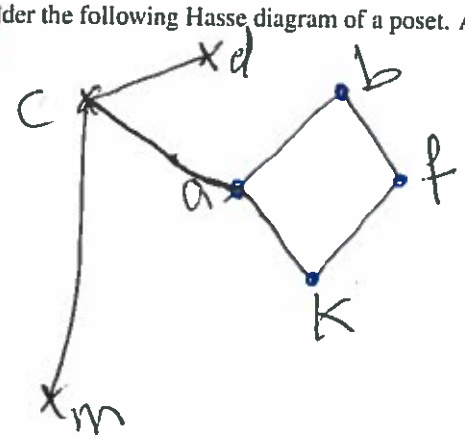


	a	b	c	d	e	f	g
a	[0^a]	3^a	5^a	6^a	∞	∞	∞
b	×	[3^a]	5^a	5^b	∞	∞	∞
c	×	×	[5^a]	5^b	11^c	8^c	12^c
d	×	×	×	[5^b]	11^c	8^c	12^c
f	×	×	×	×	11^c	[8^c]	9^f
g	×	×	×	×	11^g	×	[9^f]
e	×	×	×	×	[11^g]	×	×



b/b

QUESTION 2. (6 points) Consider the following Hasse diagram of a poset. Answer the following:



(i) $m \wedge a$ DNE ✓

(ii) $m \vee a$ c ✓

(iii) $c \vee b$ DNE ✓

(iv) $c \wedge b$ a ✓

(v) $k \wedge d$ k ✓

(vi) $k \vee b$ b ✓

b/b

QUESTION 3. Can we construct a tree with the following degrees : 4, 3, 1, 1, 1, 1, 1? Explain

4 3 1 1 1 1 1

4 3 1 1 1 1 1

2 0 0 0 1 1

2 1 1 0 0 0

v_1 v_2 v_3 v_4 v_5 v_6

2 1 1 0 0 0

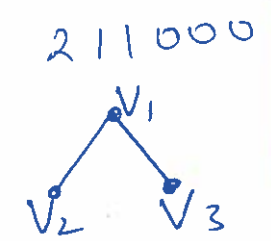
Yes, it is possible

Since 2 1 1 0 0 0

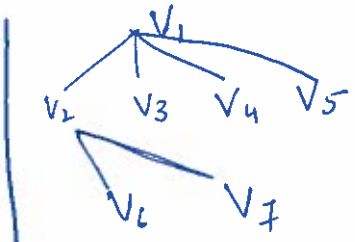
can be sketched

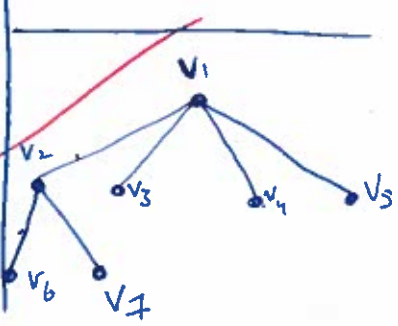
b/b

2 1 1 0 0 0



v_4 v_5 v_6





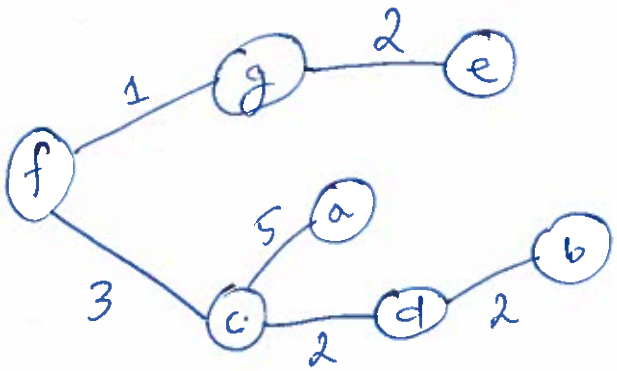
b00075480

UTK

(1)

	a	b	c	d	e	f	g
f	∞	∞	3^f	9^f	5^f	$\boxed{0^f}$	9^f
g	∞	∞	3^g	9^f	3^g		
e	∞	∞	3^f	9^f	$\boxed{3^g}$	X	$\boxed{11^f}$
c	8^c	∞	$\boxed{3^f}$	5^c	X	X	X
d	8^c	7^d	X	$\boxed{5^c}$	X	X	X
b	8^c	$\boxed{7^d}$	X	X	X	X	X
a	$\boxed{8^c}$	X	X	X	X	X	X

Minimum Spanning tree



QUESTION 4. Let $D = Z*$ (The set of all integers). Define $\leq$ on A such that for all $a, b \in D$, we have $a \leq b$ iff $a = bc$ for some $c \in Z^*$. Convince me that $\leq$ is not a poset on D.

$b \leq a$ $\quad b = ac$

$a \leq b$ ✗

$2 \leq -2, (2)$ ✗

$-2 \leq 2, (-2)$ → satisfies $a \leq b$ because $-2 = 2x-1$

$-2 = -2x1$

However, it also satisfies $b \leq a$ which is wrong $2 = 2x-1$

Therefore $a \leq b$ ✓ $\quad b \not\leq a$ → not satisfied

$2 \leq -2$ if $2 = -2x$

$2 = -2x-1$

$2 = 2$ ✓

Theo axiom
Pf $a \leq b$ then
$b \not\leq a$ fails

QUESTION 5. Let $A = Z$. Define "=" on A such that for all $a, b \in A$ we have a = "b if and only if $a - b = 6c$ for some $c \in Z$. Then "=" is an equivalence relation on A. Find all equivalence classes.

	-6	-5	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
[0]						0	α	α	α	α	α	1	-1					-2	
[1]						α	0 ✓	α	α	α	α	α	-1						2 ✓

[4]

$[0] = \{\ldots -12, -6, 0, 6, 12, \ldots\}$

$[1] = \{\ldots -5, 1, 7, 13 \ldots\}$

$[2] = \{\ldots -4, 2, 8, 14 \ldots\}$

$[3] = \{\ldots -3, 3, 9, 15 \ldots\}$

$[4] = \{\ldots -2, 4, 10, 16 \ldots\}$

$[5] = \{\ldots -1, 5, 11, 17, \ldots\}$

4-16
$-12 = 6c$
$-12 = 6x-2$
$= -12$ ✓

4/4

$a \leq a$

$a = ac$

$5-11 = 6c$
$-6 = 6x-1$ ✓

$5-(-1) = 6c$
$6 = 6x1$ ✓

$4-4 = 6c$
$0 = 6x0$

$4-10 = 6c$
$-6 = 6x-1$ ✓

QUESTION 6. Given the following function $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 1 & 5 & 2 & 3 & 8 & 6 & 7 \end{pmatrix}$

a) Find F^2

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 4 & 3 & 1 & 5 & 7 & 8 & 6 \end{pmatrix}$

1-4 1
4-2 2
2-1 3
1-4 4
4-2 5
2-1 6 ✓

b) Find the least positive integer $n \geq 1$ such that $f^n = I$.

$(1\ 4\ 2) \circ (3\ 5) \circ (6\ 8\ 7)$

LCM(3, 2, 3)

2 |	2, 3, 3
3 |	1, 3, 3
	1, 1, 1

$2 \times 3 = 6$

least positive integer = 6

QUESTION 7. (i) What is the meaning of a connected graph?

A connected graph is a graph where for each two distinct vertices, there exist a path between the two.

(ii) What is the meaning of a complete graph?

~~Between~~ Each two distinct vertices are connected through an edge

(iii) Can we construct a graph with 10 vertices such that $deg(v_1) = 7, deg(v_2) = 5, deg(v_3) = 5, deg(v_4) = deg(v_5) = deg(v_6) = ... = deg(v_{10}) = 2$? explain

7 5 5 2 2 2 2 2 2 2

4 4 1 1 1 1 1 2 2

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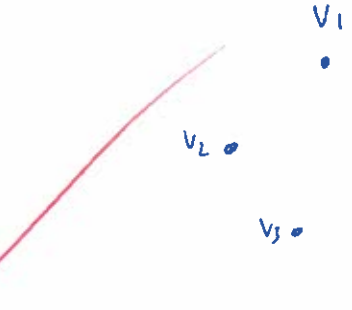
4 4 2 2 1 1 1 1 1

3 1 1 0 1 1 1 1

3 1 1 1 1 1 1 0

0 0 0 1 1 1 0

v_1	v_2	v_3	v_4	v_5	v_6	v_7
1	1	1	0	0	0	0



not possible to sketch this

thus the graph with the 10 vertices cannot be sketched because 1 of the first 3 vertices gets a degree 2